Although white noise is a useful abstraction, no noise process can truly be white; however, the noise encountered in many real systems can be assumed to be approximately white. We can only observe such noise after it has passed through a real system which will have a finite bandwidth. Thus, as long as the bandwidth of the noise is appreciably larger than that of the system, the noise can be considered to have an infinite bandwidth.

The delta function in Equation (1.43) means that the noise signal, $n(t)$, is totally decorrelated from its time-shifted version, for any $\tau > 0$. Equation (1.43) indicates that *any* two different samples of a white noise process are uncorrelated. Since thermal noise is a Gaussian process and the samples are uncorrelated, the noise samples are also independent [3]. Therefore, the effect on the detection process of a channel with *additive white Gaussian noise* (AWGN) is that the noise affects each transmitted symbol *independently*. Such a channel is called a *memoryless channel*. The term "additive" means that the noise is simply superimposed or added to the signal—that there are no multiplicative mechanisms at work.

Since thermal noise is present in all communication systems and is the prominent noise source for most systems, the thermal noise characteristics—additive, white, and Gaussian—are most often used to model the noise in communication systems. Since zero-mean Gaussian noise is completely characterized by its *variance*, this model is particularly simple to use in the detection of signals and in the design of optimum receivers. In this book we shall assume, unless otherwise stated, that the system is corrupted by *additive zero-mean white Gaussian noise*, even though this is sometimes an oversimplification.

1.6 SIGNAL TRANSMISSION THROUGH LINEAR SYSTEMS

Having developed a set of models for signals and noise, we now consider the characterization of systems and their effects on such signals and noise. Since a system can be characterized equally well in the time domain or the frequency domain, techniques will be developed in both domains to analyze the response of a linear system to an arbitrary input signal. The signal, applied to the input of the system, as shown in Figure 1.9, can be described either as a time-domain signal, $x(t)$, or by its Fourier transform, $X(f)$. The use of time-domain analysis yields the time-domain output, $y(t)$, and in the process, $h(t)$, the characteristic or *impulse response* of the network, will be defined. When the input is considered in the frequency domain, we shall define a *frequency transfer function*, $H(f)$, for the system, which will determine the frequency-domain output, $Y(f)$. The system is assumed to be linear and time invariant. It is also assumed that there is no stored energy in the system at the time the input is applied.

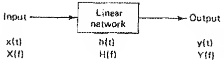

Figure 1.9 Linear system and its key parameters.

1.6.1 Impulse Response

The linear time-invariant system or network illustrated in Figure 1.9 is characterized in the time domain by an impulse response, $h(t)$, which is the response when the input is equal to a unit impulse $\delta(t)$; that is,

$$h(t) = y(t) \qquad \text{when } x(t) = \delta(t) \tag{1.45}$$

The response of the network to an arbitrary input $x(t)$ is then found by the convolution of $x(t)$ with $h(t)$, where $*$ denotes the convolution operation (see Section A.5):

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)\, d\tau \tag{1.46}$$

The system is assumed to be *causal,* which means that there can be *no* output prior to the time, $t = 0$, when the input is applied. Therefore, the lower limit of integration can be changed to zero, and we can express the output $y(t)$ as

$$y(t) = \int_{0}^{\infty} x(\tau)h(t - \tau)\, d\tau \tag{1.47}$$

Equations (1.46) and (1.47) are called the *superposition integral* or the *convolution integral.*

1.6.2 Frequency Transfer Function

The frequency-domain output signal, $Y(f)$, is obtained by taking the Fourier transform of both sides of Equation (1.46). Since convolution in the time-domain transforms to multiplication in the frequency domain (and vice versa), Equation (1.46) yields

$$Y(f) = X(f)H(f) \tag{1.48}$$

or

$$H(f) = \frac{Y(f)}{X(f)} \tag{1.49}$$

provided, of course, that $X(f) \neq 0$ for all f. Here $H(f) = \mathcal{F}\{h(t)\}$, the Fourier transform of the impulse response function, is called the *frequency transfer function* or the *frequency response* of the network. In general, $H(f)$ is complex and can be written as

$$H(f) = |H(f)|\, e^{j\theta(f)} \tag{1.50}$$

where $|H(f)|$ is the magnitude response. The phase response, $\theta(f)$, is defined as

$$\theta(f) = \tan^{-1} \frac{\text{Im}\,\{H(f)\}}{\text{Re}\,\{H(f)\}} \tag{1.51}$$

where the terms "Re" and "Im" denote "the real part of" and "the imaginary part of," respectively.

The frequency transfer function of a linear time-invariant network can easily be measured in the laboratory with a sinusoidal generator at the input of the network and an oscilloscope at the output. When the input waveform $x(t)$ is expressed as

$$x(t) = A \cos 2\pi f_0 t$$

the output of the network will be

$$y(t) = A\,|H(f_0)| \cos [2\pi f_0 t + \theta(f_0)] \tag{1.52}$$

The input frequency, f_0, is stepped through the values of interest; at each step, the amplitude and phase at the output are measured.

1.6.2.1 Random Processes and Linear Systems

If a random process forms the input to a time-invariant linear system, the output will also be a random process. That is, each sample function of the input process yields a sample function of the output process. The input power spectral density, $G_X(f)$, and the output power spectral density, $G_Y(f)$, are related as follows:

$$G_Y(f) = G_X(f)\,|H(f)|^2 \tag{1.53}$$

Equation (1.53) provides a simple way of finding the power spectral density out of a time-invariant linear system when the input is a random process.

In Chapters 2 and 3 we consider the detection of signals in Gaussian noise. We will utilize a fundamental property of a Gaussian process applied to a linear system, stated as follows: It can be shown that if a Gaussian process, $X(t)$, is applied to a time-invariant linear filter, the random process, $Y(t)$, developed at the output of the filter is also Gaussian [6].

1.6.3 Distortionless Transmission

What is required of a network for it to behave like an *ideal* transmission line? The output signal from an ideal transmission line may have some time delay compared to the input, and it may have a different amplitude than the input (just a scale change), but otherwise it must have no distortion—it must have the same shape as the input. Therefore, for ideal distortionless transmission, we can describe the output signal as

$$y(t) = Kx(t - t_0) \tag{1.54}$$

where K and t_0 are constants. Taking the Fourier transform of both sides (see Section A.3.1), we write

$$Y(f) = KX(f)e^{-j2\pi f t_0} \tag{1.55}$$

Substituting the expression (1.55) for $Y(f)$ into Equation (1.49), we see that the required system transfer function for distortionless transmission is

$$H(f) = Ke^{-j2\pi f t_0} \tag{1.56}$$

Therefore, to achieve *ideal distortionless transmission*, the overall system response must have a constant magnitude response, and its phase shift must be linear with frequency. It is not enough that the system amplify or attenuate all frequency components equally. All of the signal's frequency components must also arrive with identical time delay in order to add up correctly. Since time delay, t_0, is related to phase shift, θ, and radian frequency, $\omega = 2\pi f$, as follows,

$$t_0 \text{ (seconds)} = \frac{\theta \text{ (radians)}}{2\pi f \text{ (radians/second)}} \tag{1.57}$$

it is clear that phase shift must be proportional to frequency in order for the time delay of all components to be identical. In practice, a signal will be distorted in passing through some parts of a system. Phase or amplitude correction (*equalization*) networks may be introduced elsewhere in the system to correct for this distortion. It is the overall input–output characteristic of the system that determines its performance.

1.6.3.1 Ideal Filter

One cannot build the ideal network described in Equation (1.56). The problem is that Equation (1.56) implies an infinite bandwidth capability, where the bandwidth of a system is defined as the interval of positive frequencies over which the magnitude $|H(f)|$ remains within a specified value. In Section 1.7 various measures of bandwidth are enumerated. As an approximation to the ideal infinite-bandwidth network, let us choose a truncated network that passes, without distortion, all frequency components between f_ℓ and f_u, where f_ℓ is the lower cutoff frequency and f_u is the upper cutoff frequency, as shown in Figure 1.10. Each of these networks is called an *ideal filter*. Outside the range $f_\ell < f < f_u$, which is called the *passband*, the ideal filter is assumed to have a response of zero magnitude. The effective width of the passband is specified by the filter bandwidth $W_f = (f_u - f_\ell)$ hertz.

When $f_\ell \neq 0$ and $f_u \neq \infty$, the filter is called a *bandpass filter* (BPF), shown in Figure 1.10a. When $f_\ell = 0$ and f_u has a finite value, the filter is called a *low-pass filter* (LPF), shown in Figure 1.10b. When f_ℓ has a nonzero value and when $f_u \rightarrow \infty$, the filter is called a *high-pass filter* (HPF), shown in Figure 1.10c.

Following Equation (1.56), for the ideal low-pass filter transfer function with bandwidth $W_f = f_u$ hertz, shown in Figure 1.10b, we can write the transfer function as follows (letting $K = 1$):

$$H(f) = |H(f)|\, e^{-j\theta(f)} \tag{1.58}$$

where

$$|H(f)| = \begin{cases} 1 & \text{for } |f| < f_u \\ 0 & \text{for } |f| \geq f_u \end{cases} \tag{1.59}$$

and

$$e^{-j\theta(f)} = e^{-j2\pi f t_0} \tag{1.60}$$